

of the mollusca inhabiting the mountain country, and the molluscan fauna of the great delta of the Ganges and Brahmaputra had its origin in the Himalayan slopes, although they have occupied their present quarters for sufficiently long a period to become specifically distinct. (BV)².

WATER AND THE PUBLIC HEALTH.

- (1) *The Value of Pure Water.* By George C. Whipple. Pp. viii+84. (New York: John Wiley and Sons; London: Chapman and Hall, Ltd., 1907.) Price 4s. 6d. net.
- (2) *The Bacteriological Examination of Water Supplies.* By Dr. William G. Savage. Pp. xvi+297. (London: H. K. Lewis, 1906.) Price 6s. 6d. net.

(1) THIS little book is planned on novel lines and deserves recognition. An extract will best serve to define its scope.

"Given two water sources equally available to a city for purposes of supply, both safe to drink, but one high coloured and soft, the other colourless and hard—which is the better selection? A water-works plant is to be appraised: structurally the system is a good one, but the quality of the water is unsatisfactory because of its excessive colour or turbidity—how much should be deducted from the value of the works because of the bad quality of the water? The water-works owned by a private company are to be purchased by the city; the city has a high typhoid fever death-rate, due unquestionably to the water supply—how much less should the city pay because of that fact? A city is using turbid river water—how much can it afford to pay to filter it? A city is using a water so heavily laden with Anabæna that it is nauseous to drink—how much can the city afford to pay to procure a new supply?"

An attempt is made from available data to establish formulæ which may be employed to calculate the allowable depreciation due to sanitary quality, physical characteristics (colour, odour, &c.), hardness, &c., of a water supply.

The following example is a calculation in the case of typhoid fever. The financial loss to the community for each death from typhoid fever is assumed from various data to be equal to 10,000 dollars. A proportion of the death-rate is due to the disease transmitted by means (shell-fish, flies, &c.) other than water. Assuming that all typhoid in excess of N is due to the water supply, that the daily consumption of water is 100 gallons per capita, and that T is the total typhoid death-rate per 100,000, then (T-N) 10,000=loss to the community in dollars for $365 \times 100 \times 100,000$ gallons of water, or

$$D = \frac{(T - N)10000}{365} = 2.75(T - N),$$

where D stands for the loss in dollars per million gallons of water used. The author is quite alive to the fact that local and other conditions must modify his conclusions, and recognises that more data are required before finality is reached in the elaboration of the formulæ. The book is suggestive and stimulating reading, the

various tables add to its value, and we heartily commend it to the sanitarian and water engineer.

(2) This book by Dr. Savage, who has made many important contributions to the subject on which he writes, gives an admirable survey of the present position of the bacteriology of water supplies. Successive chapters deal with the bacterial content of waters and the influences affecting it, excreta, sewage, and soil in relation to the bacteriological examination, the characters of the intestinal bacteria, bacterial evidences of pollution, and full details of the methods employed in the bacteriological examination of water. The chapter on the interpretation of results is particularly to be recommended. A full bibliography is appended. The medical officer of health, the analyst, and the bacteriologist will find this book a trustworthy and useful guide.

R. T. HEWLETT.

THREE MATHEMATICAL TRACTS.

Quadratic Forms and their Classification by Means of Invariant Factors. By Prof. T. J. I'A. Bromwich, F.R.S. Pp. viii+100. (Cambridge: The University Press, 1906.) Price 3s. 2d. net.

The Axioms of Projective Geometry. By Dr. A. N. Whitehead, F.R.S. Pp. viii+64. (Cambridge: The University Press, 1906.) Price 2s. 6d.

The Axioms of Descriptive Geometry. By Dr. A. N. Whitehead, F.R.S. Pp. viii+74. (Cambridge: The University Press, 1907.) Price 2s. 6d.

THESE are Nos. 3, 4, and 5 of the Cambridge Tracts in Mathematics and Mathematical Physics, which are intended to help students by providing them with brief and readable introductions to mathematical theories which are important in themselves, and yet for various reasons do not appear in the ordinary text-books. If they serve their purpose they will induce their readers to follow up the paths they indicate, and try to explore still further the mazy garden of the mathematical muse.

The present state of the theory of quadratic forms illustrates very well how much interest there may be in the particular cases of a problem which, in its so-called "general form," has a trite and familiar solution. To put the matter into a geometrical shape; when there are four homogeneous variables, let $S=0$, $T=0$ be the equations of two quadratic surfaces; then in general the family $S+\lambda T=0$ will have a common self-conjugate tetrahedron, and by taking this as a tetrahedron of reference, S and T assume a well-known standard form. But there are thirteen other cases to consider, for each of which there is a distinct reduced form of $S+\lambda T$; for instance, if S and T intersect in a cuspidal quartic, the reduced form is

$$2(\lambda + a)xy + 2yz + b(\lambda + a)z^2 + c(\lambda + d)t^2.$$

If we consider the small oscillations of a dynamical system with four degrees of freedom, we are confronted by precisely the same analytical problem of reduction; the algebraical classification is the same, but certain cases are ruled out by the condition that

the kinetic energy of the system is necessarily positive; still more, if the potential energy is a true minimum.

Prof. Bromwich has given an excellent account of the analytical theory, with various geometrical and dynamical illustrations, and he has added a very useful bibliography. As he has pointed out himself, he has selected Kronecker's method of proof of the invariance of the factors of the discriminant of $S + \lambda T$; and he has made no reference to the specially arithmetical form of the problem, where the coefficients of the forms are integers, and the equations of transformation have to be unitary and integral. For this, the student will consult Frobenius, Hensel, and H. J. Smith, whose memoirs, of course, Prof. Bromwich includes in his list of references.

Dr. Whitehead's chapters deal with a subject which, on the one hand, is more ancient than that of Prof. Bromwich, but, on the other, has changed its aspect recently in a much more remarkable way. Dr. Whitehead is one of the company of sappers who are reducing all the mathematical part of geometry to a system of abstract logic applied to a minimum number of undefinable entities, connected by a minimum number of undefinable relations. Put in this bald way, their work seems purely destructive and hateful, but in reality it is not so. In the tract on projective geometry it is shown how, with the help of Dedekind's axiom, and those of order, it is possible to make rigorous von Staudt's proof that all the points on a line are either reached by harmonic constructions starting from three given points, or definable as limiting points of a set of such points. This leads to definitions of numerical cross-ratios and of numerical homogeneous coordinates which are independent of any theory of distance or measurement, a very remarkable and far-reaching result. It is very encouraging to find that the magnificent genius of von Staudt is gradually gaining the recognition that it deserves; the interval between him and his predecessors is at least as great as that between Apollonius and Steiner.

By "descriptive" geometry Dr. Whitehead means "any geometry in which two straight lines in a plane do not necessarily intersect." Besides the discussion of preliminary axioms and definitions, his second tract falls into two principal parts; the first deals with the problem of enlarging a descriptive space into a projective space (the simplest example is that of adjoining the plane at infinity to Euclidean space), the second with the theory of displacements and measurement. The latter is based upon, what is, perhaps, the only satisfactory method—that of Sophus Lie. The last chapter gives the formulæ of metrical geometry in the shape given to them by Cayley and Laguerre, so that, neglecting a constant numerical factor, a distance and an angle are each measured by the logarithm of a cross-ratio. The cross-ratio, of course, must be *projectively* defined, otherwise we should be in a vicious circle, and it is in the avoidance of this circle that the latest perfection of the theory consists.

G. B. M.

OUR BOOK SHELF.

Practical Agricultural Chemistry. By F. D. S. Robertson. Pp. x+210. (London: Baillière, Tindall and Cox, 1907.) Price 7s. 6d. net.

IN his introduction the author tells us that his book is intended as a course of laboratory practice for the use of students in agricultural chemistry. How much time does he suppose such students can give to agricultural chemistry to allow them to deal with subjects like the valuation of tea and coffee, or the determination of hop resin and glycerin in beer? To what class of readers is a paragraph like the following of use? "*The Bitter Used.* This is necessarily a tedious operation, and for full particulars the reader is referred to such books upon poisons as describe the processes of Dragendorff and others. The prepared and concentrated beer is subjected to a series of extractions with petroleum ether, benzene, chloroform, and amyl alcohol, each of which is examined in turn."

Even in the more properly agricultural parts of the book there is little evidence that the author possesses any working acquaintance with his subject, e.g. the chapter on the analysis of soils is the merest skeleton, possessing small reference to the methods in regular use, and containing actual errors, such as the attempt to estimate humus by solution in ammonia without a preliminary treatment of the soil with acid.

Again, in his description of the Reichert-Wollny process for estimating volatile acids in butter, the author says nothing of the official standardisation of the dimensions of the apparatus and other details, which, however, must be followed if figures are to be obtained comparable with those of other analysts, and indeed are absolutely essential if the analyst is doing public work. We cannot recommend Mr. Robertson's book.

An Episode of Flatland, or How a Plane Folk discovered the Third Dimension, to which is added An Outline of the History of Unaea. By C. H. Hinton. Pp 181. (London: Swan Sonnenschein and Co., Ltd.)

THESE plane people live on the edge of a disc which is their world. The third dimension exists only in their mathematics. Their astronomers find that a catastrophe will certainly happen. One cranky philosopher believes that there is a third dimension, and shows a scared people how their world may be tilted and the catastrophe averted. The author's characters act and make love much like three-dimensional people, and they talk of a higher dimension just as Mr. Hinton would himself talk of a fourth dimension. There never was an allegory, not even that of Bunyan, which was consistent with itself for one chapter, but Mr. Hinton's is more inconsistent with itself than any other allegory we have seen. J. P.

The Bernese Oberland. Vol. iii. Dent de Morcles to the Gemmi. By H. Dübi. Pp. xxiv+136. (London: T. Fisher Unwin, 1907.) Price 10s.

THIS conveniently arranged pocket-book is the most recent addition to the "Climbers' Guides" edited by Sir Martin Conway and the Rev. W. A. B. Coolidge. The southern limit of the region described is the Rhone Valley from Martigny to Leuk; and the northern is marked by the low passes leading from the Ormonts-Lessus glen to Kandersteg by the heads of the Grande Eau, the Sarine, the Simme, and the Kander valleys, which mark it off from the foothills. The preface of the book directs attention to the fact that the present is the jubilee year of the formation of the Alpine Club and of the ascents of the Oldenhorn and the Wildstrubel, and we are confident that the increase in the number of climbers during the last fifty years will ensure a wide popular for this workmanlike volume of "marching orders."